

AN INITIAL INVESTIGATION INTO METHODS OF COMPUTING  
TRANSONIC AERODYNAMIC SENSITIVITY COEFFICIENTS

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Semiannual Progress Report  
January 1991 -- June 1991

TEXAS A&M UNIVERSITY

TAMRF Report No. 5802-91-02

July 1991

NASA Grant No. NAG 1-793

Leland A. Carlson  
Professor of Aerospace Engineering  
Texas A&M University  
College Station, TX 77843-3141

TEXAS ENGINEERING EXPERIMENT STATION

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## I. Introduction

This report covers the period from January 1991 thru June 1991. During this reporting period, work has continued on studies associated with the development of the "quasi-analytical" sensitivity method for three dimensional transonic flow about wings. Further, initial results using the quasi-analytical approach have been obtained and compared to those computed using the finite difference approach.

## II. Personnel

The individuals associated with the project during this reporting period have been Dr. Leland A. Carlson, Principal Investigator, and Hesham Elbanna, Graduate Research Assistant. Mr. Elbanna has been primarily supported by the project during this period and will use the results of this research effort for his Ph.D. dissertation.

## III. Research Progress

During the past six months, considerable progress has been made in developing the quasi-analytical approach to obtain aerodynamic sensitivity coefficients about wings in transonic flow. These efforts and some preliminary subcritical and supercritical results are summarized by a report prepared by Mr. Elbanna and contained herein as Appendix I. As can be seen from the appendix, most of the problems encountered during the last reporting period have been solved; and the quasi-analytical approach now yields for basic design variables accurate values for subcritical cases and reasonable values for supercritical conditions.

As can be seen in the supercritical results in Appendix I, there is a slight disagreement between the quasi-analytical and the finite-difference approaches in the vicinity of the shock. This "discrepancy" appears usually as to the location of the "jump" in sensitivity coefficients. Since the preparation of the appendix, this disagreement has been traced to the treatment of the subsonic-supersonic switch function in the quasi-analytical approach. Initial efforts to correct this problem, while still being checked, appear to eliminate the difficulty and lead to excellent agreement between finite-difference and quasi-analytical results at both subcritical and supercritical conditions.

Again, for further details concerning the present research status of the project, the reader is referred to Appendix I.

#### IV. Project Status

During this period, additional funds were awarded to the Grant to cover the period 1 January 1991 thru 31 December 1991. Since these funds were not actually received until March, it is anticipated that a request to extend the present grant period appropriately will be submitted in the near future.

#### V. Future Efforts

During the next six months, work will continue on developing the quasi-analytical approach and verifying its usefulness from a proof-of-concept viewpoint. In addition, the design variables considered will be extended to include planform variables. Further, the entire method will be extended to include supersonic freestream conditions.

## VI. Technical Monitor

The technical monitor for this project is Dr. E. Carson Yates, Jr., Interdisciplinary Research Office, NASA Langley Research Center.

**APPENDIX I**

**Determination of Aerodynamic Sensitivity Coefficients  
Based on the Full Potential Formulation**

**H. M. Elbanna**

**July 1991**

## Nomenclature

C	Maximum camber
CG	Conjugate gradient
Cl	Local lift coefficient
CL	Total lift coefficient
Cp	Pressure coefficient
FD, QA	Finite-difference, quasianalytical methods
GMRES	Generalized minimum residual
L	Location of maximum camber
M	Freestream Mach number
XD	Vector of design variables
T	Maximum thickness
T1,T2,T3,T4	Section twist angles
$\alpha$	Angle of attack

## Introduction

This progress report covers the period from January 1991 to July 1991. The basic goals achieved during this period are (a) Carrying out various debugging operations pertaining to the quasianalytical method, (b) Addition of section design variables to the sensitivity equation in the form of multiple right hand sides, (c) Reconfiguring the analysis/sensitivity package in order to facilitate the execution of analysis/FD/QA test cases, and (d) Enhancing the display of output data to allow careful examination of the results and to permit various comparisons of sensitivity derivatives obtained using the FD/QA methods to be conducted easily and quickly.

In addition to discussing the above goals, this progress report includes the results (basically pressure coefficient sensitivity derivatives obtained via both the QA and FD methods) of executing subcritical and supercritical test cases. It is to be noticed that details of the development of the analysis/sensitivity package including various MACSYMA, FORTRAN, and DISPLAY programs, are found in previous reports<sup>1,2,3</sup> and therefore only a brief discussion of the latest debugging effort is outlined in the following section. Final documentation of all programs will be turned in at a later stage.

## Debugging Operations

Various errors were detected and corrected in the MACSYMA codes. These codes, as mentioned previously<sup>1,2,3</sup>, generate FORTRAN source code for the jacobian matrix and right hand side vectors. Among the important factors realized were (a) the need to use flag variables to signal different wing/wake boundary points, (b) the inclusion of the wing symmetry boundary condition, and (c) careful evaluation of the jacobian terms related to the wing boundary condition and the addition of these terms to those obtained from the general residual expression (i.e. the derivatives of the residual expression at an interior point).

In carrying out the above corrections, two main ideas were implemented. First, a separate MACSYMA code was quickly put together to test the evaluation of various residual terms. This step was very helpful in directing the search for errors and quickly revealed deficient variables. Second, a return to the two-dimensional sensitivity code showed a programming error in handling the three-dimensional wing boundary condition. Accordingly, this error was corrected and the wing boundary condition was handled similar to the way the airfoil boundary condition was set up in the two-dimensional program.

### Design Variables

Currently, the vector of design variables for the three-dimensional problem includes,

- (a) Freestream design variables: freestream Mach number and the angle of attack.
- (b) Cross-section design variables: maximum thickness, maximum camber, location of maximum camber, and section twist angles at given input spanwise stations.

Accordingly, the results presented herein are the sensitivities of the pressure coefficient  $C_p$  (obtained via both the QA and FD methods) with respect to the vector of design variables  $XD$  where,

$$XD = [M, \alpha, T, C, L, T1, T2, T3, T4]$$

### Test Cases and Results

Two ONERA M6 wing planform test cases are presented. The first case is for a NACA-1406 airfoil section at a freestream Mach number of 0.8 and at an angle of attack of one degree (this is a subcritical test case). The second case is for the same airfoil section at a Mach number of 0.84 and at an angle of attack of three degrees (supercritical test case).

For the subcritical case, comparison of the results obtained by the quasianalytical method show excellent agreement with the results obtained via the finite-difference method. Furthermore, these results follow the trend of the two-dimensional problem<sup>4</sup>. As for the supercritical test case, a few comments are in order. First, both methods show slight disagreement at and about the shock location. Currently, this disagreement is being investigated. Second, the finite-difference method exhibits accuracy and oscillation problems as can be seen from Figures 2,6,7,8, and 9. Third, the results, again and in general, follow the trend of those of the two-dimensional study<sup>4</sup>. In addition to the above, an overall comparison of the values of the integrated coefficients at each spanwise station and obtained using both the FD and QA methods (these values are displayed next to the legend on each figure and are also presented on the last page of each test case as a spanwise distribution) show good agreement and are in general less in magnitude than the corresponding values for the two-dimensional problem, which is to be expected.

It is to be noticed that in both of the above cases, a coarse/medium grid is used in computing the analysis information. This grid sequencing approach has the advantage of speeding up convergence. In regard to the FD method, the sensitivity information is computed by restarting each of the perturbed design states from the coarse grid then proceeding to the medium grid. Different strategies for grid sequencing together with various choices of a suitable starting solution to speed up the FD approach will be examined later on. As for the QA method, the medium grid  $45 \times 30 \times 16$  gives rise to a large, sparse, banded, and unsymmetric jacobian matrix of size about  $17500 \times 17500$ . An incomplete LU factorization is applied only once to this large matrix, and the sensitivity equation is solved using two efficient library solvers<sup>5</sup>. These library routines are based on the iterative CG and GMRES methods<sup>6,7</sup> and achieve high performance by using "the state-of-the-art" algorithms<sup>5</sup> tailored to specific operational characteristics of the IBM-3090 hardware, such as cache size, vector section size, number of vector registers available, and page size. Following the factorization of the jacobian matrix, back substitution using the known right hand side vectors generates the unknown sensitivity derivatives, with a trivial computational cost. Notice and recall that one crucial objective of this work will be to exploit the competitiveness of the QA method as the number of design variables is increased.

Currently, the cost of obtaining the sensitivity derivatives using the FD method is about 1300 CPU seconds ( $130 \times 10$  for nine design variables) while that of applying the QA method is about 1200 CPU seconds (about 700 seconds for evaluating the jacobian and right hand sides, and about 500 seconds for solving the sensitivity equation using the library solver) for the same vector of design variables and using the conjugate gradient linear solver. It is to be noticed that the potential exists for reducing the cost associated with the FD method (e.g. to execute the perturbed runs directly on the medium grid with the design point solution obtained on the coarse grid as a starting solution) and that associated with the QA method (e.g. to speed up the evaluation of the jacobian and right hand sides). As mentioned in the following section, this task will be an objective in future work. Therefore, the given values should only be considered as rough estimates for the price of applying each method.



### Future Work

Future work will focus on complete debugging of the QA method and on determining the sensitivity derivatives with respect to planform design variables. This includes augmenting the right hand sides of the sensitivity equation with the appropriate additional planform vectors. Typically, these vectors will be obtained by differentiating the residual expression with respect to various planform variables. Notice that this step might require some sort of semi-analytical treatment since the metrics (the quantities that involve the planform design variables) are tailored according to the wing surface thru the three-dimensional grid. In addition, minor modifications will be added to the analysis/sensitivity program in order to allow execution of supersonic test cases. Finally, various case studies will be conducted in order to improve the overall accuracy and efficiency of the QA and FD methods followed by a thorough examination, comparison, and interpretation of the results.

### References

1. Carlson, L.A., An Initial Investigation into Methods of Computing Transonic Aerodynamic Sensitivity Coefficients, TAMRF Report No.5802-89-03, December 1989.
2. Carlson, L.A., An Initial Investigation into Methods of Computing Transonic Aerodynamic Sensitivity Coefficients, TAMRF Report No.5802-90-01, July 1990.
3. Carlson, L.A., Elbanna, H.M., Determination of Aerodynamic Sensitivity Derivatives Based on the Full Potential Equation, TAMRF Report No.5802-91-01, February 1991.
4. Elbanna, H.M. and Carlson, L.A., "Determination of Aerodynamic Sensitivity Coefficients Based on the Transonic Small Perturbation Formulation", *Journal of Aircraft*, Vol 27, No.6, June 1990, pp 507-515.
5. IBM Engineering and Scientific Subroutine Library, Guide and Reference, Rel 3, SC23-0184-3.
6. Saad, Y. and Schultz, M.H., "GMRES: A Generalized Minimum Residual Algorithm for Solving Non-symmetric Linear Systems", *SIAM Journal of Scientific and Statistical Computing*, Vol.7, No.3, 1986, pp 856-869.
7. Sonnewald, Wesseling, and De Zeeuv, Multigrid and Conjugate Gradient Methods as Convergence Acceleration Techniques in Multigrid Methods for Integral and Differential Equations, pp 117-167, Edited by Paddon, D.J. and Holstein, M., Oxford University Press (Clarendon), Oxford.

# TEST CASE 1

SUBCRITICAL CASE

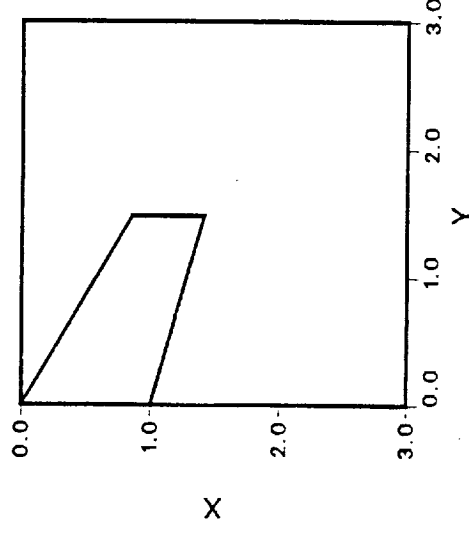
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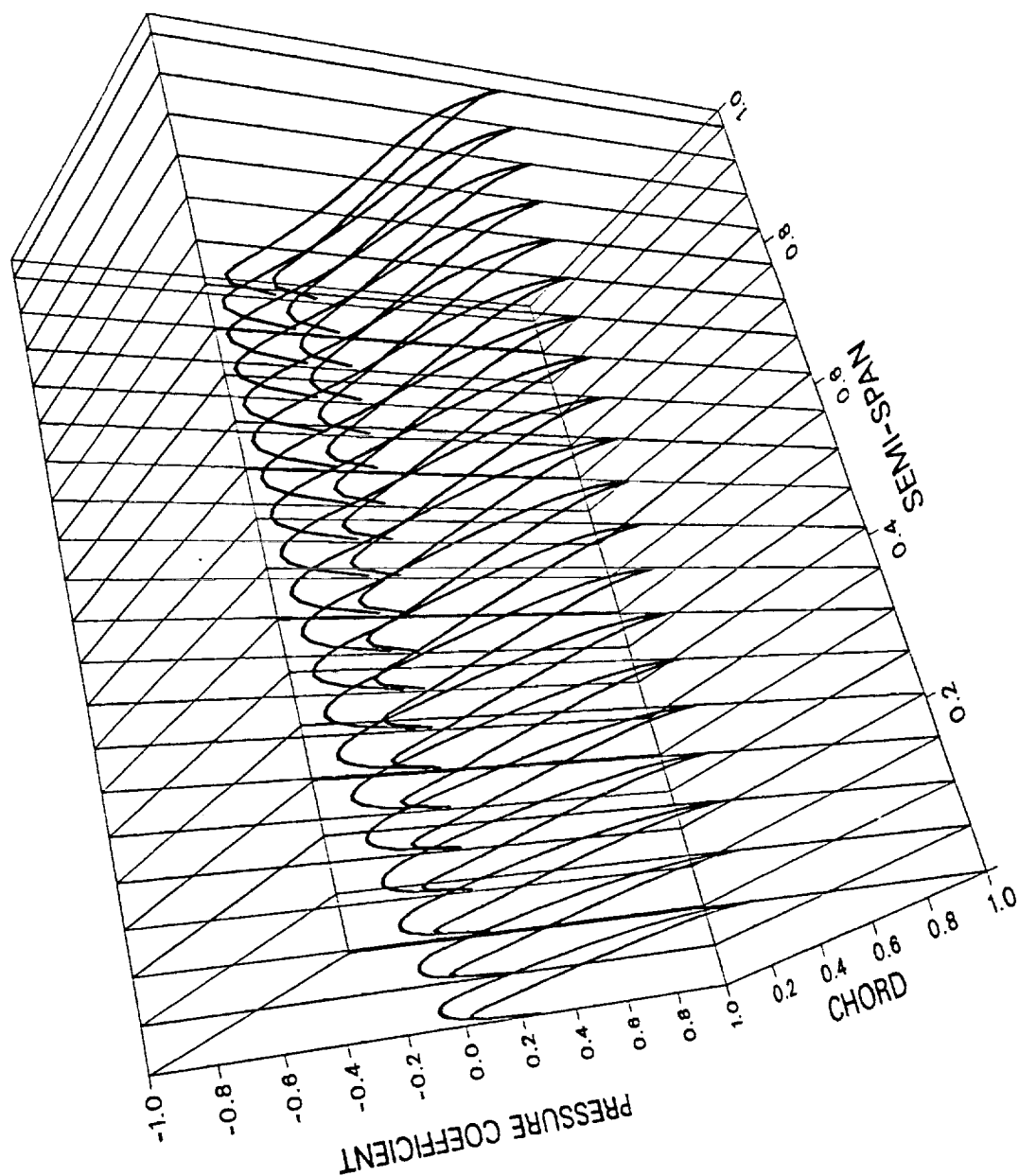
MACH NUMBER	0.80
ANGLE OF ATTACK	1.00
AIRFOIL MAX THICKNESS	0.06
AIRFOIL MAX CAMBER	0.01
LOCATION OF MAX CAMBER	0.40

# WING PLANFORM :

## ONERA M6



ROOT CHORD	1.00	ASPECT RATIO	3.80
TIP CHORD	0.56	TAPER RATIO	0.56
MEAN CHORD	0.80	SEMI SPAN	1.48
AREA	1.16	L.E. SWEEP	30.00
REF. AREA	1.16	T.E. SWEEP	15.76
REF. CHORD	0.80	ROOT TWIST	0.00
REF. MOMENT	0.25	TIP TWIST	0.00



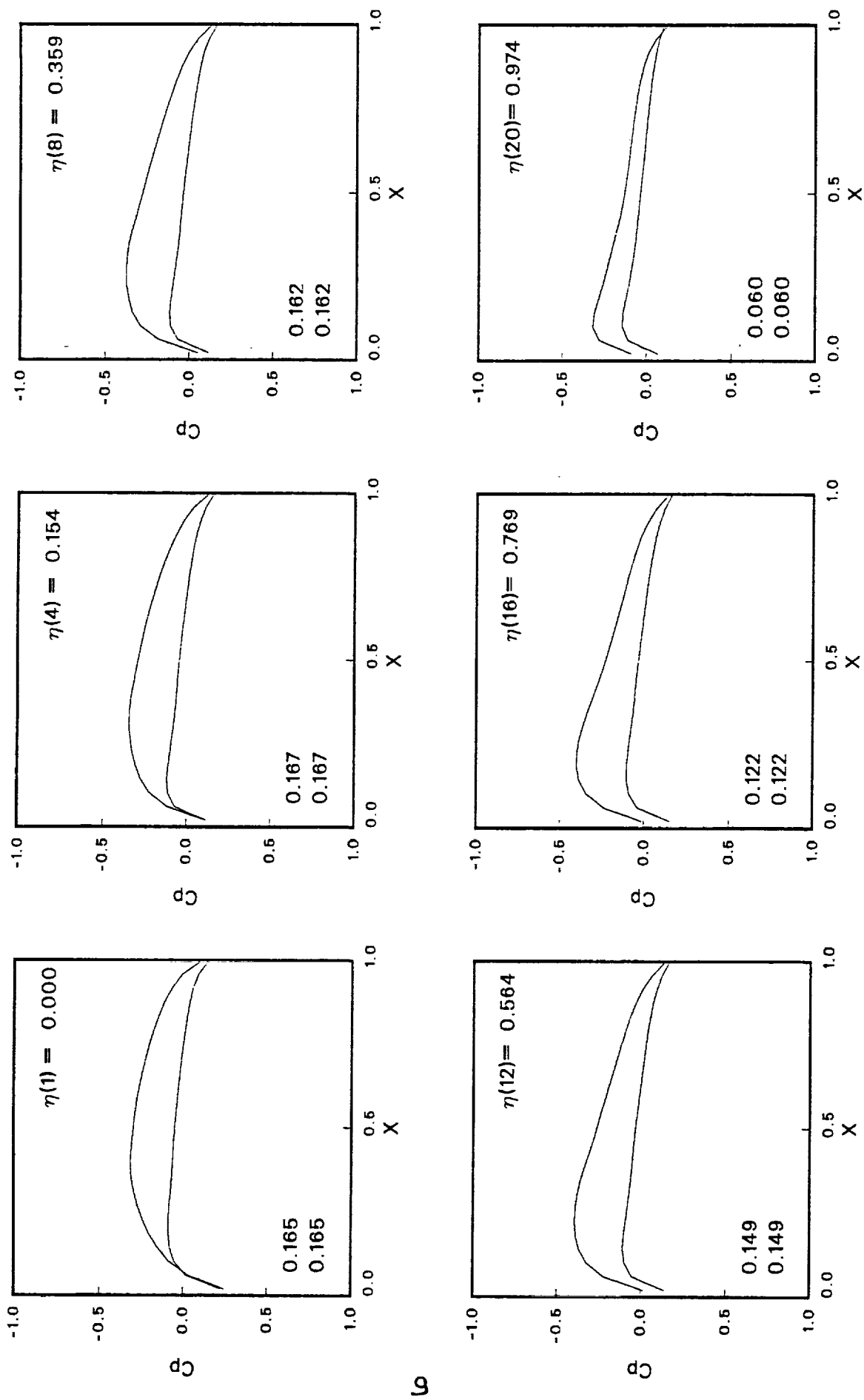


Fig.0 Pressure Coefficient

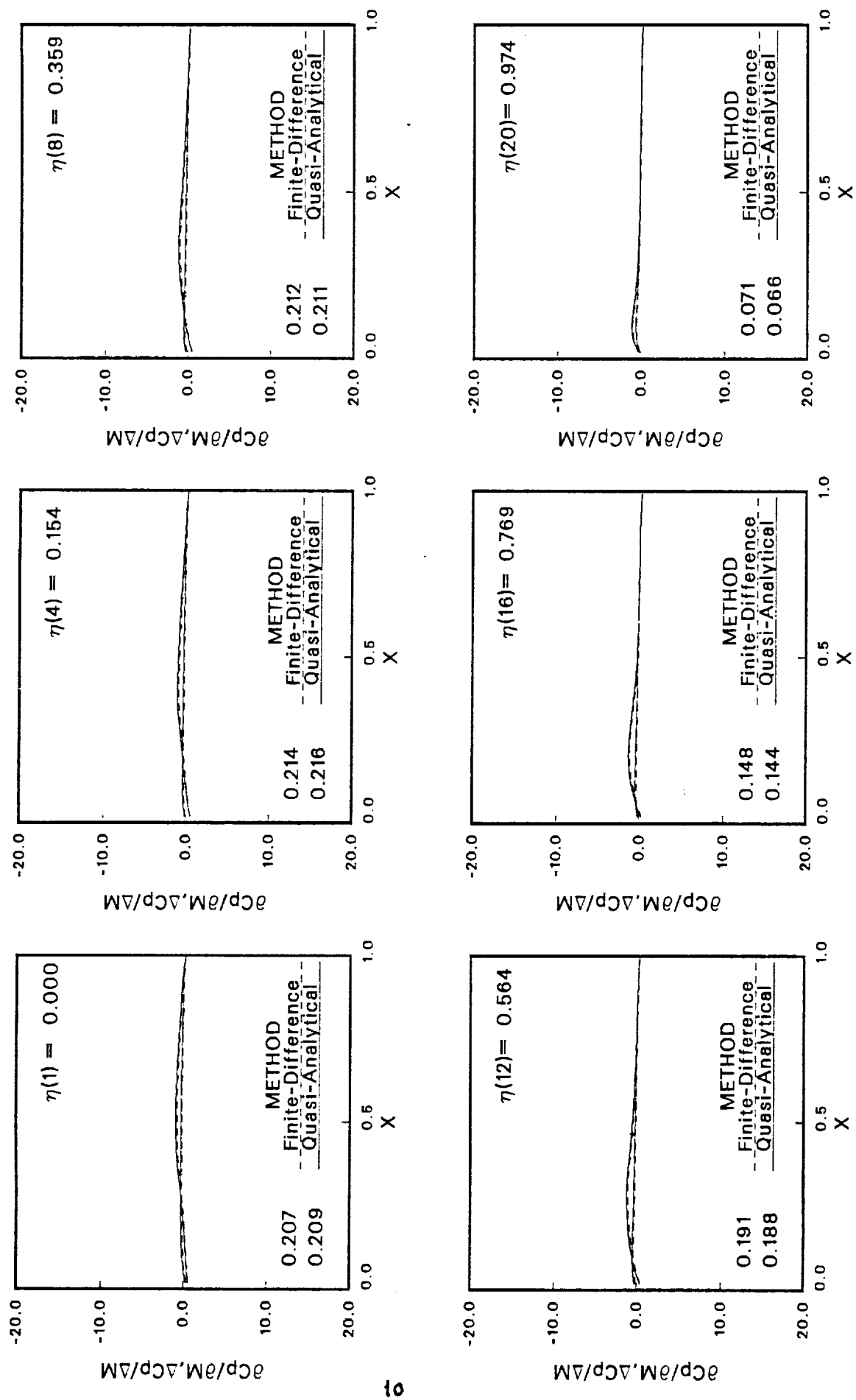


Fig.1 Sensitivity of Pressure Coefficient to Mach Number

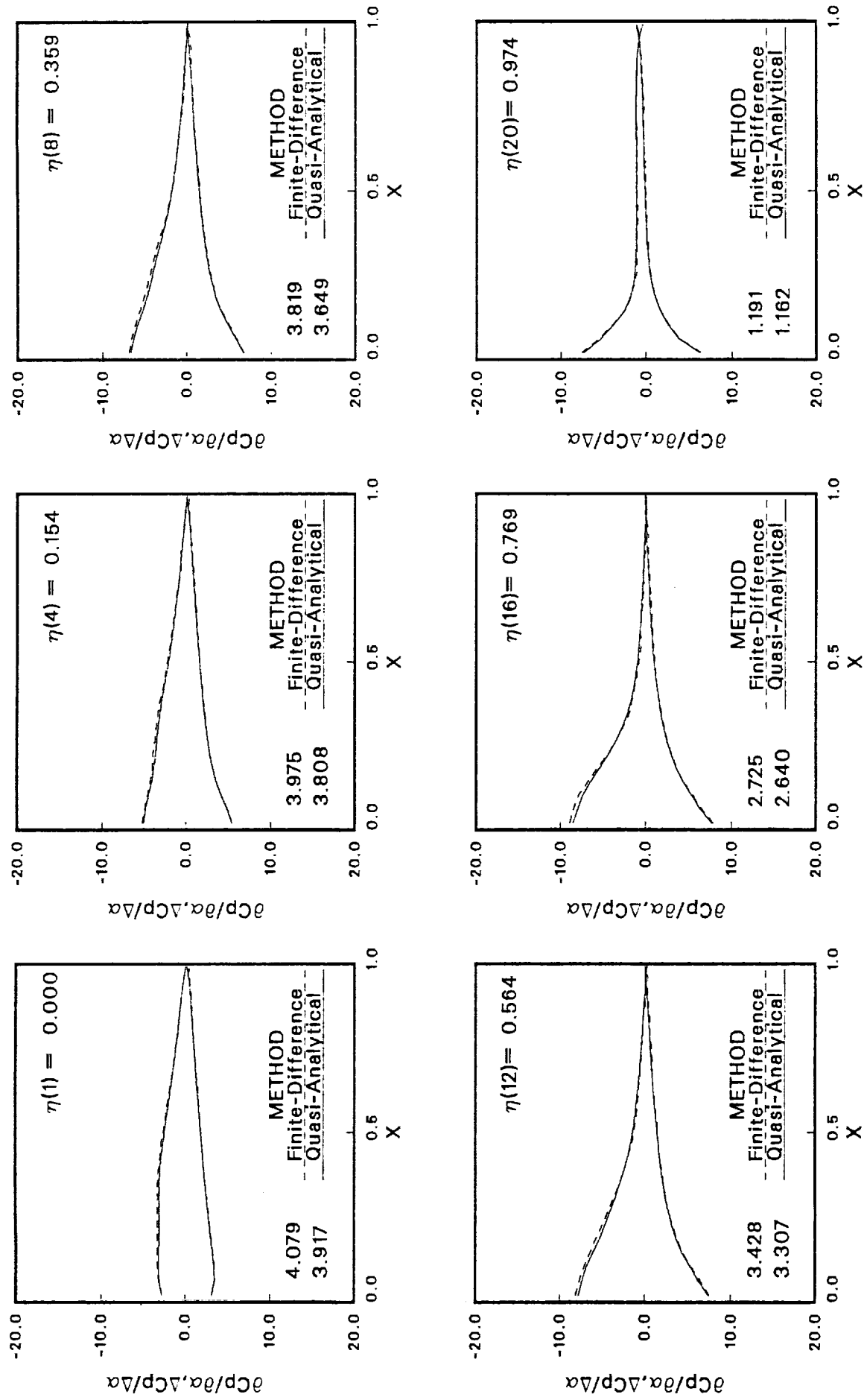


Fig.2 Sensitivity of Pressure Coefficient to Angle of Attack



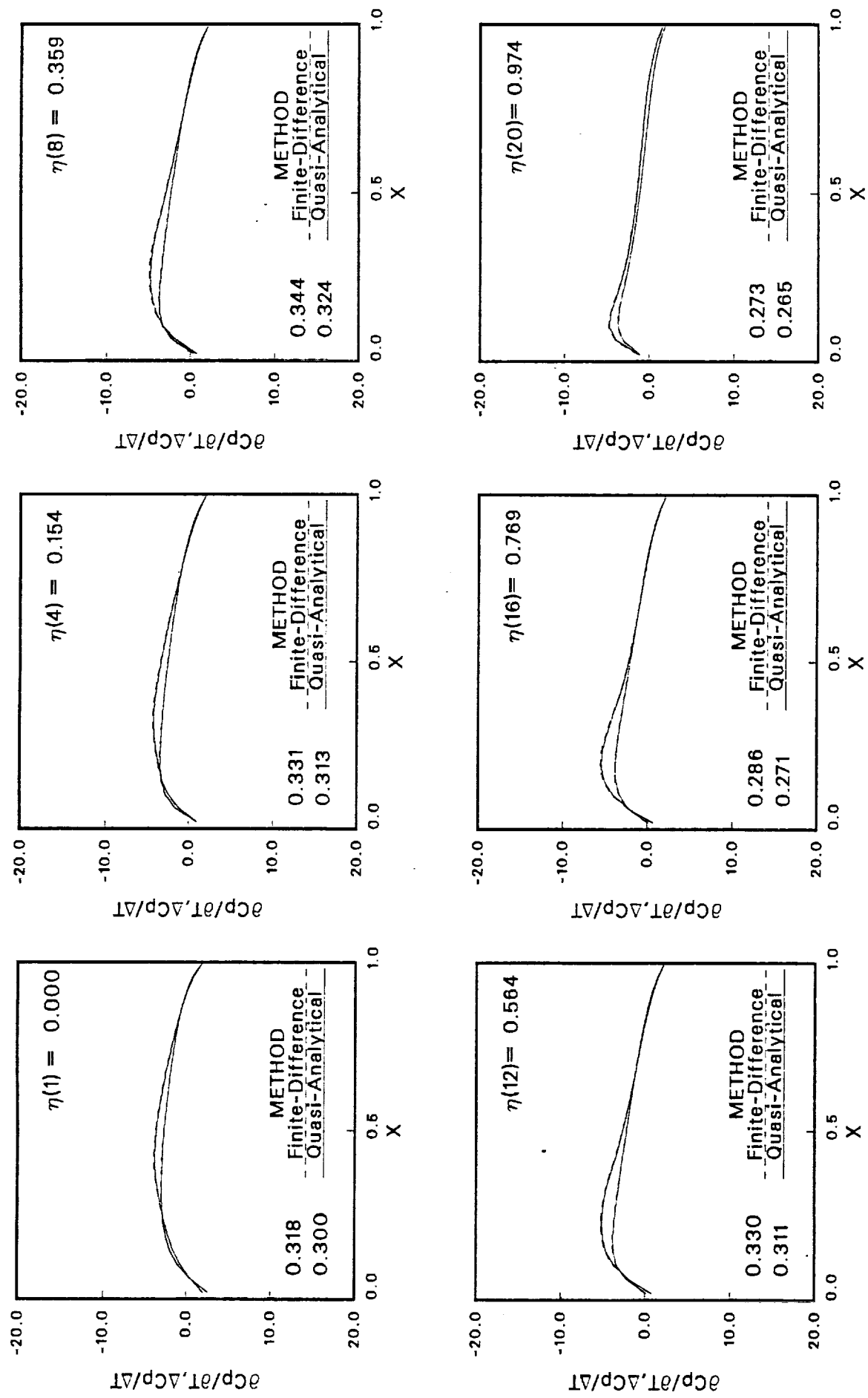


Fig.3 Sensitivity of Pressure Coefficient to Maximum Thickness

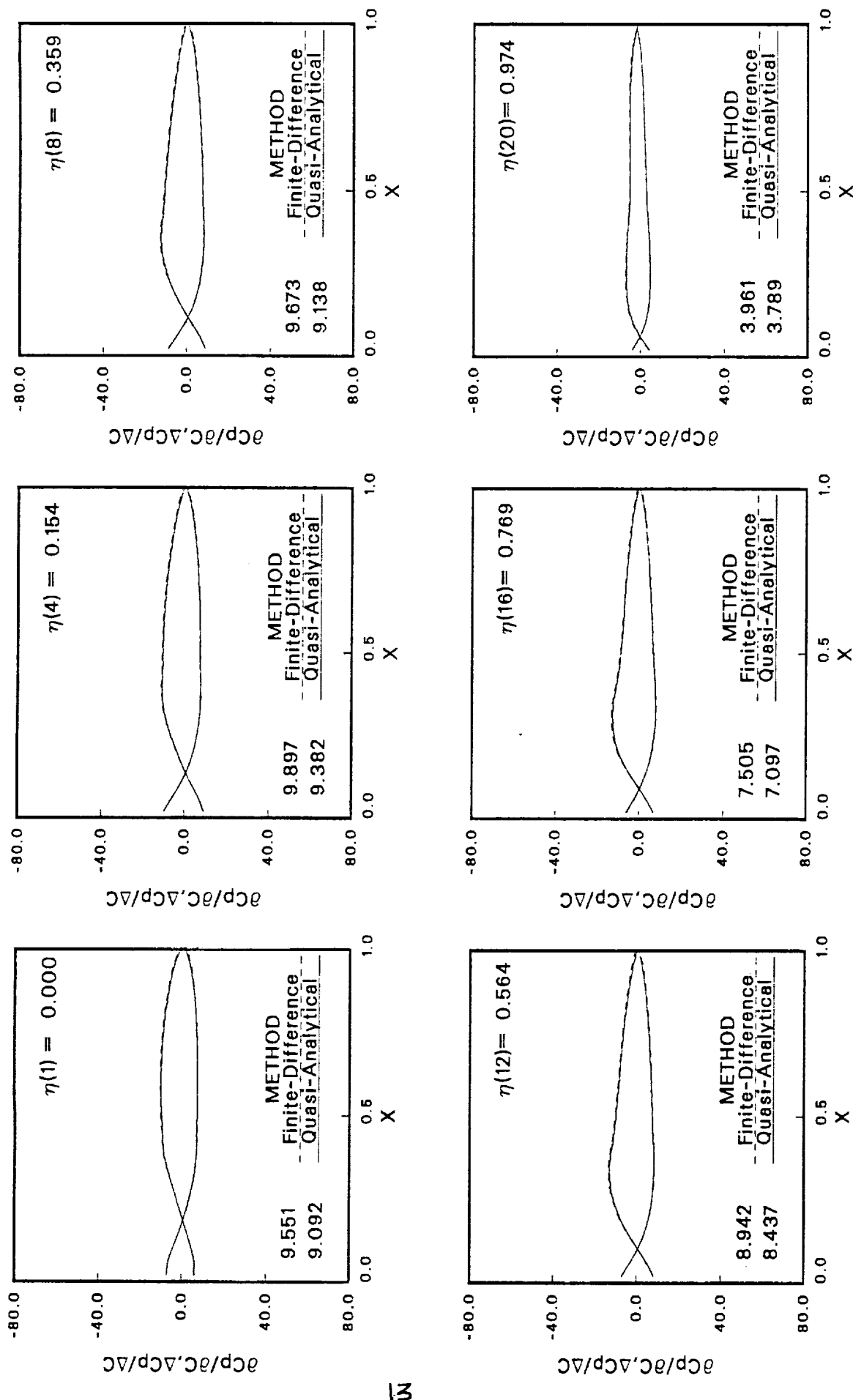


Fig.4 Sensitivity of Pressure Coefficient to Maximum Camber

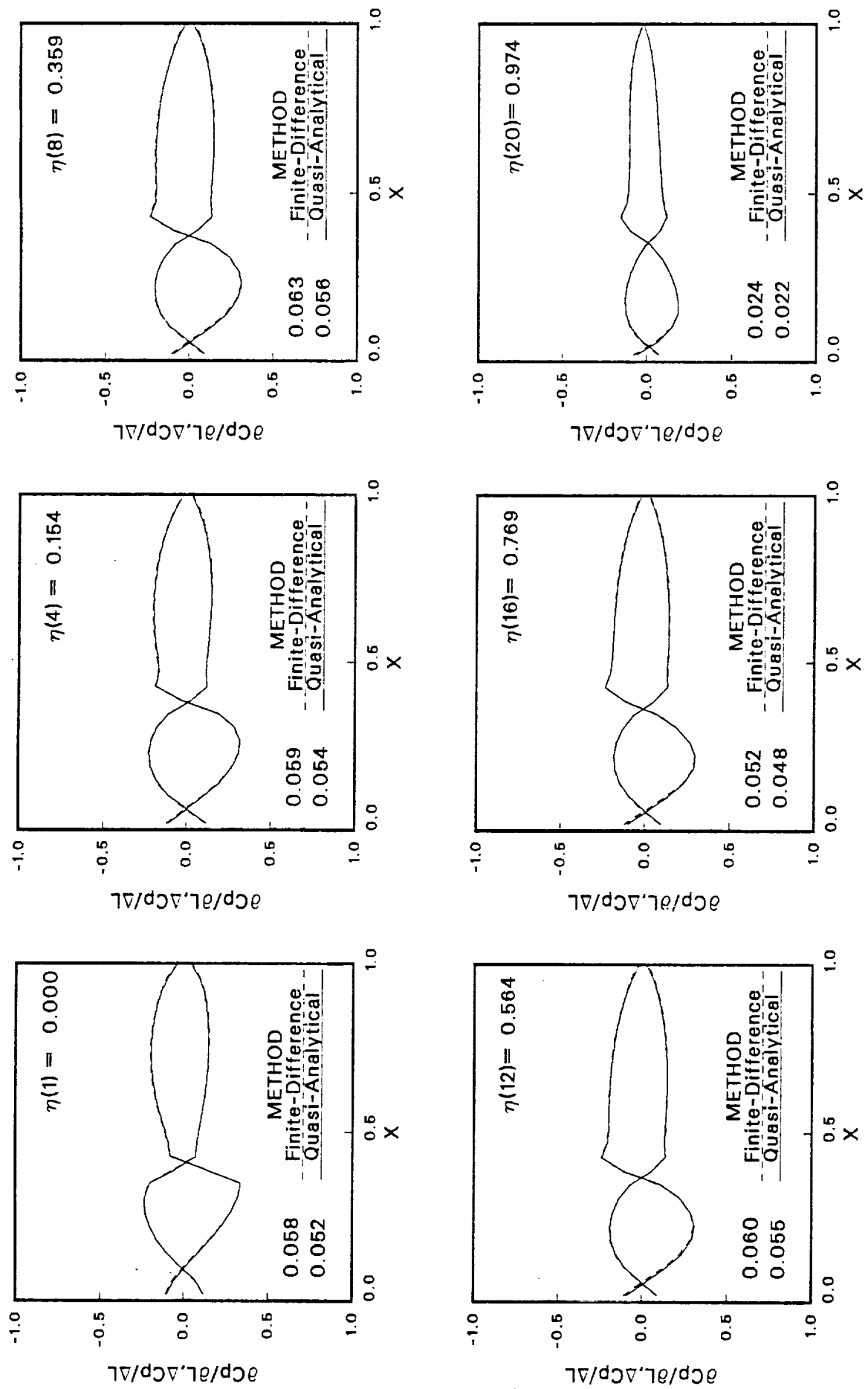


Fig.5 Sensitivity of Pressure Coefficient to Location of Maximum Camber

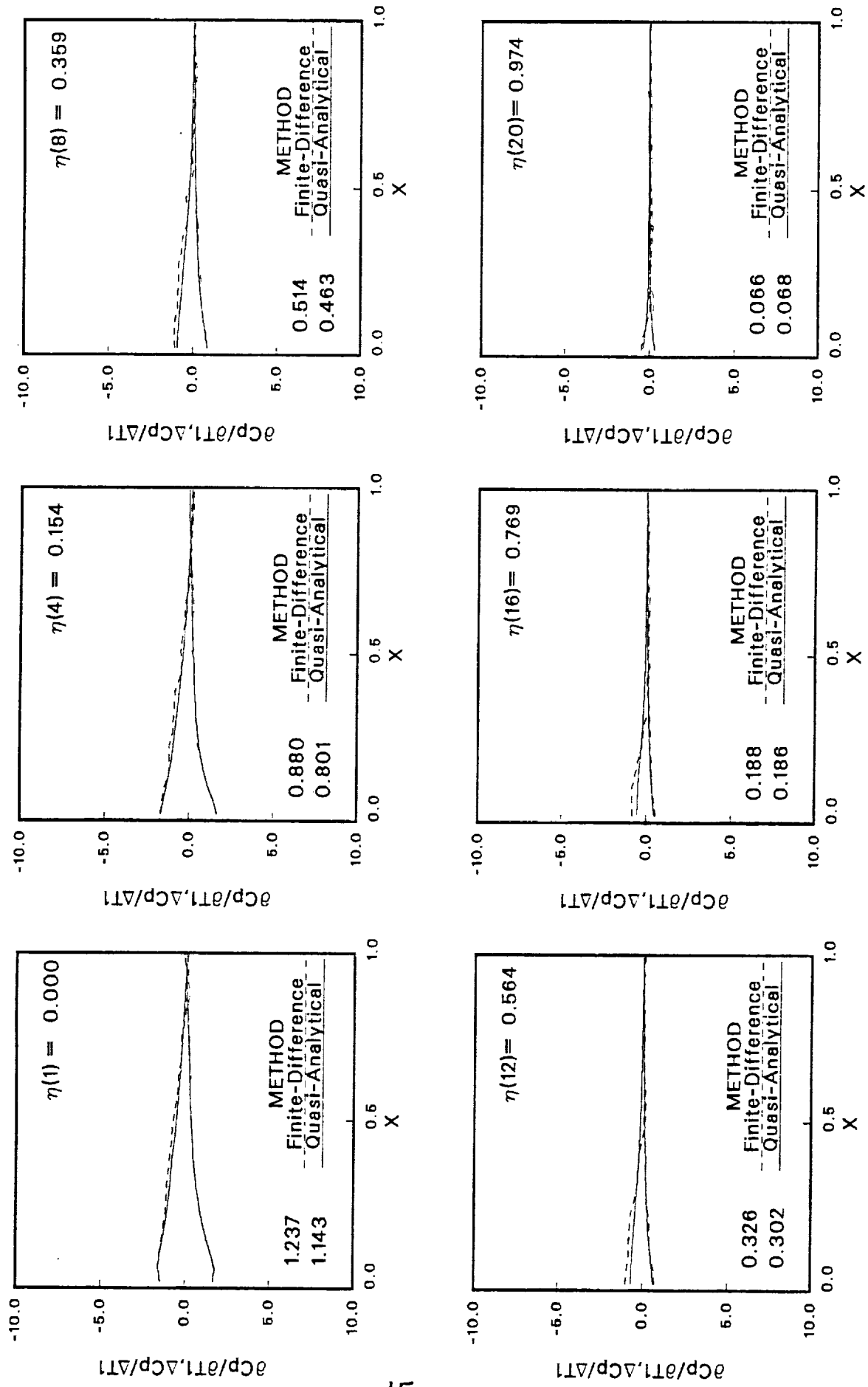


Fig.6 Sensitivity of Pressure Coefficient to Twist Angle at 1st Station

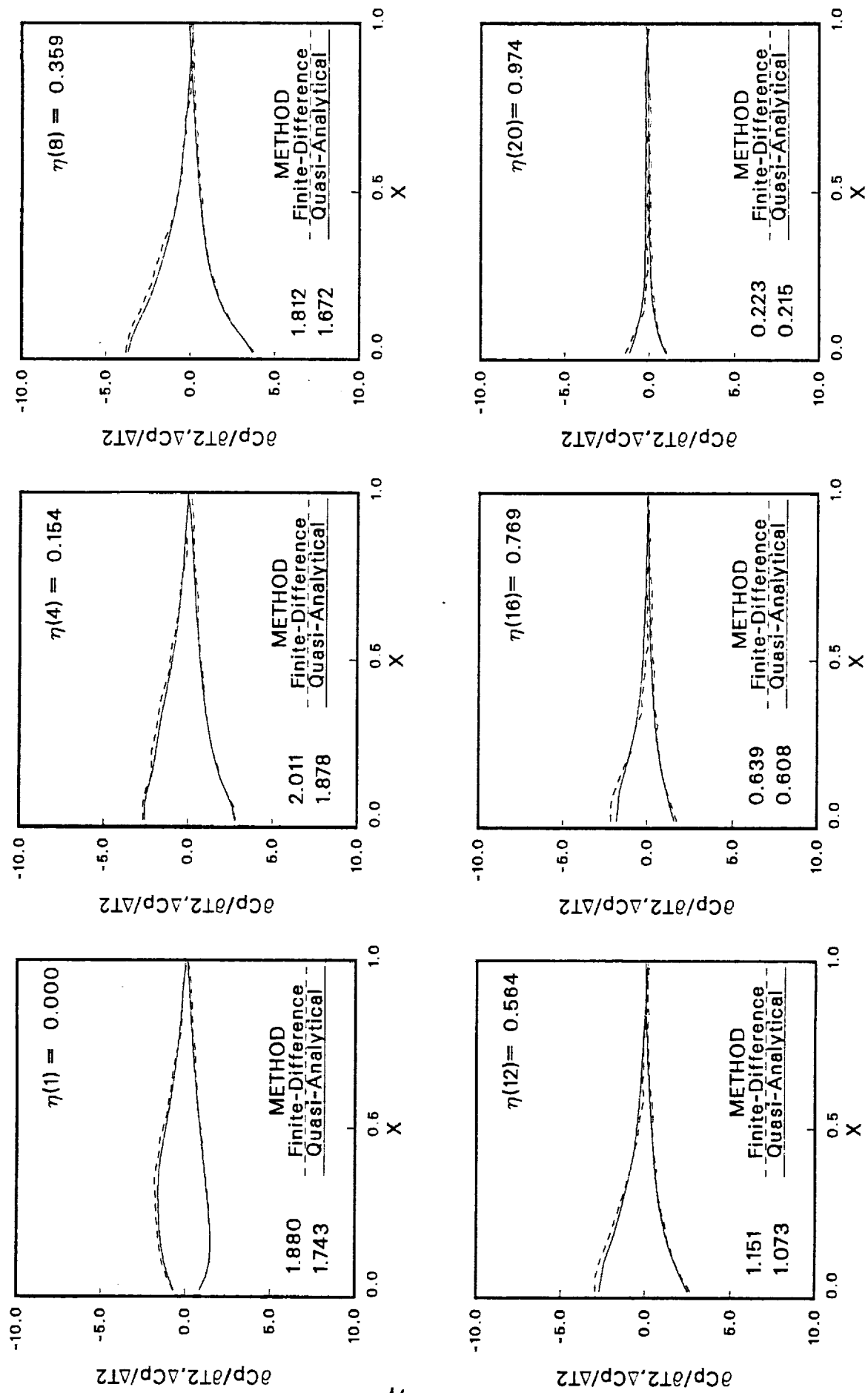


Fig.7 Sensitivity of Pressure Coefficient to Twist Angle at 2nd Station

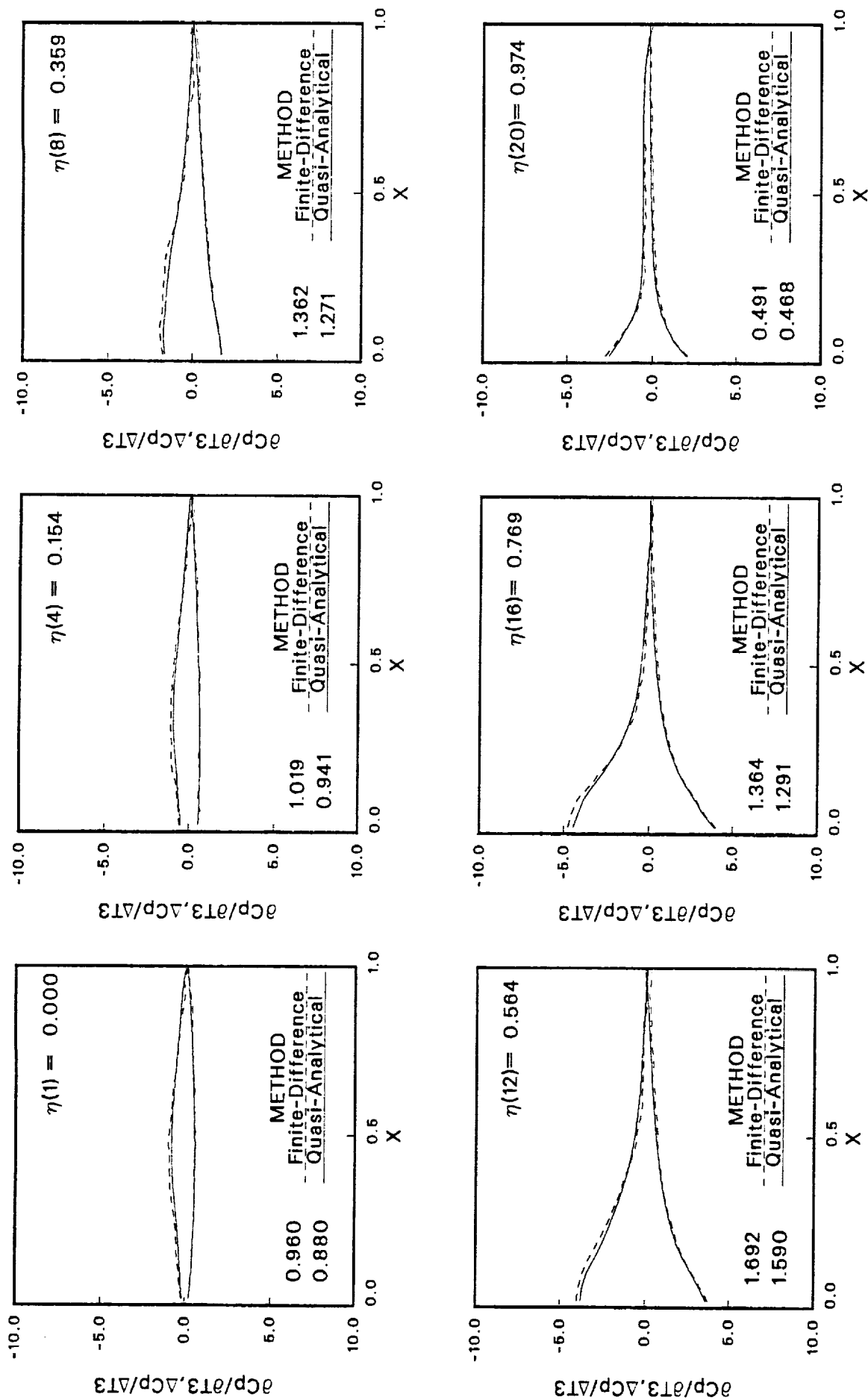


Fig.8 Sensitivity of Pressure Coefficient to Twist Angle at 3rd Station

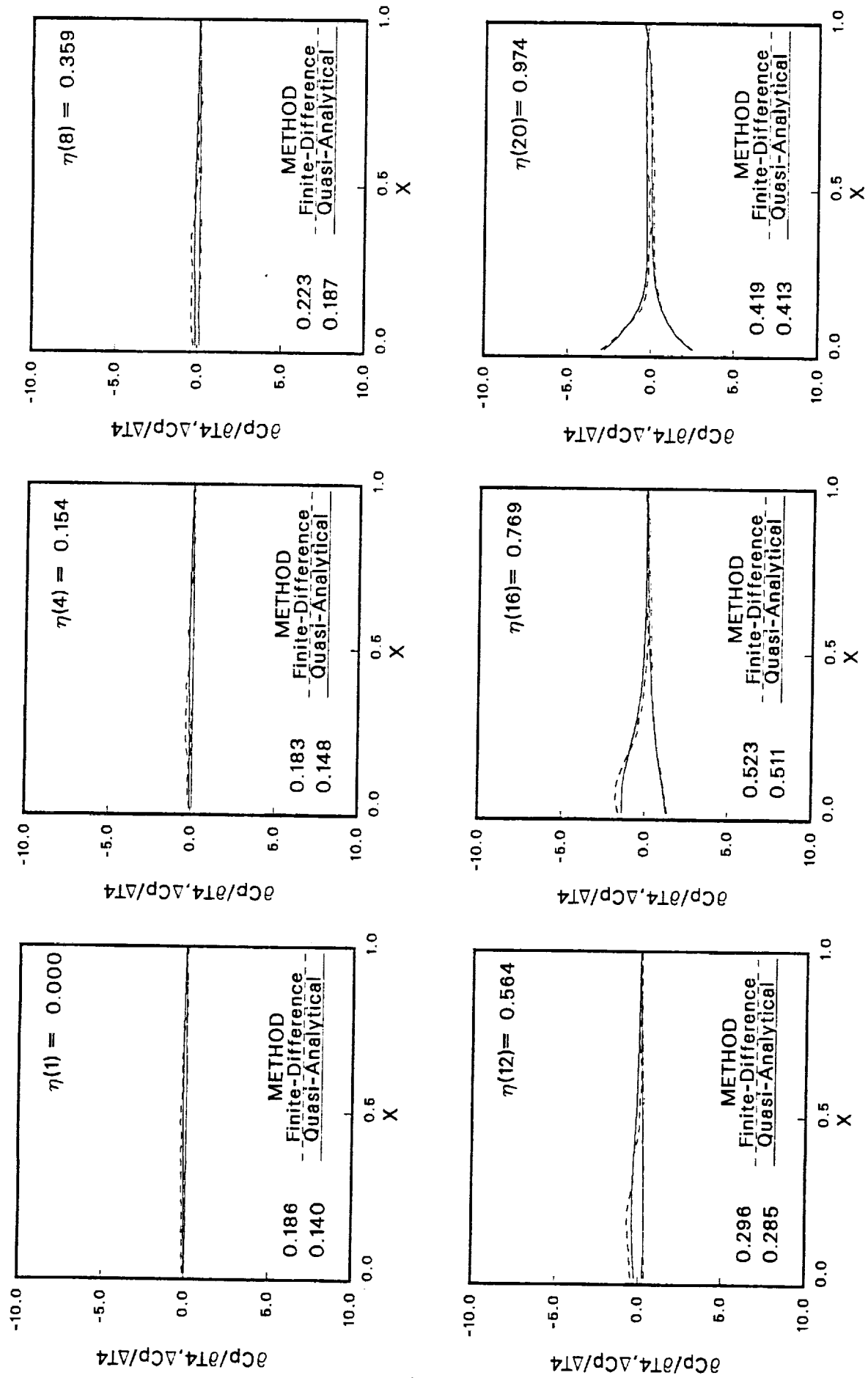
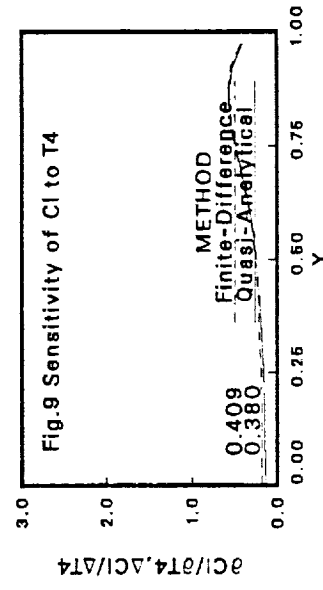
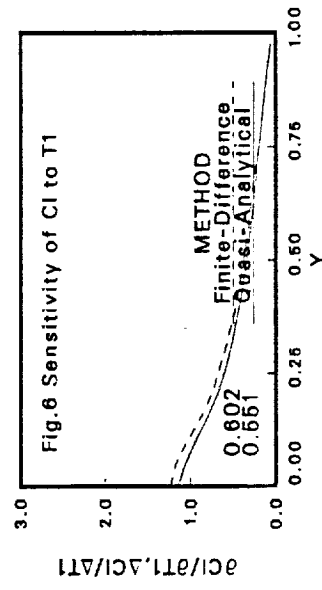
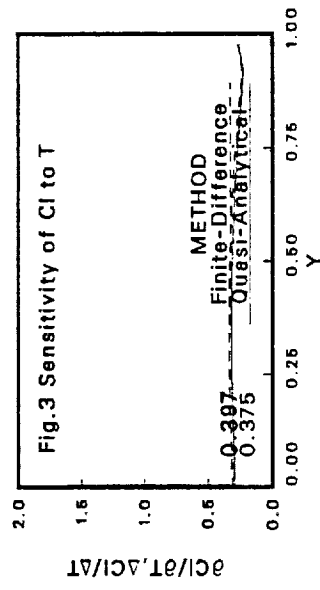
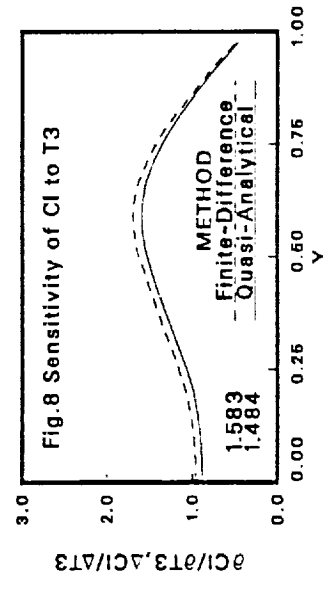
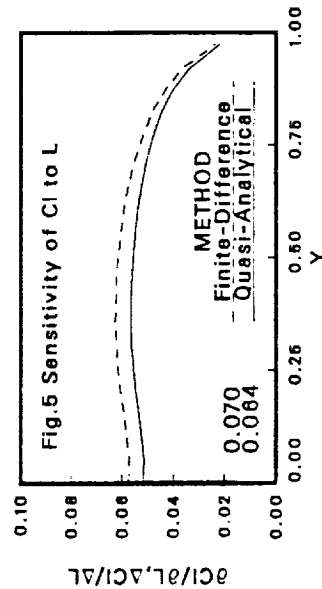
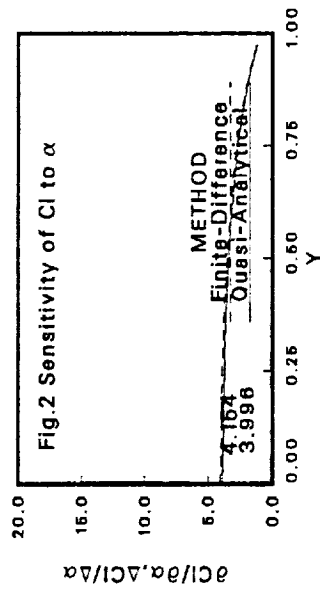
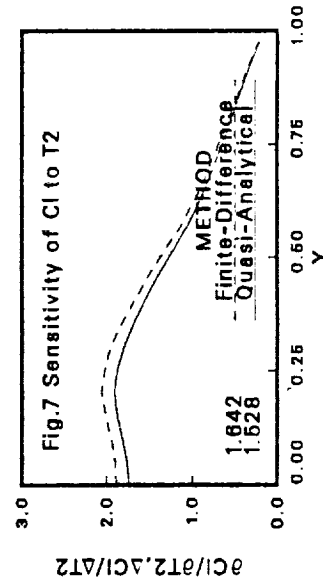
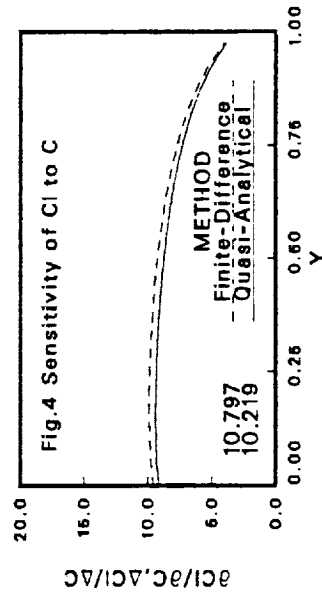
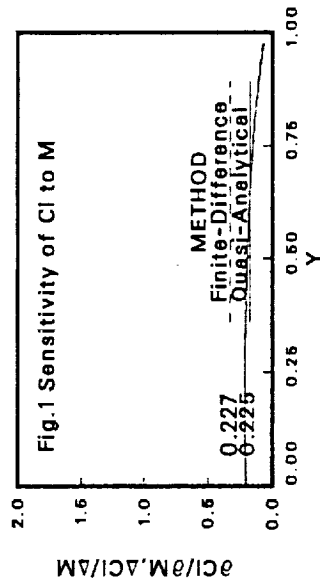
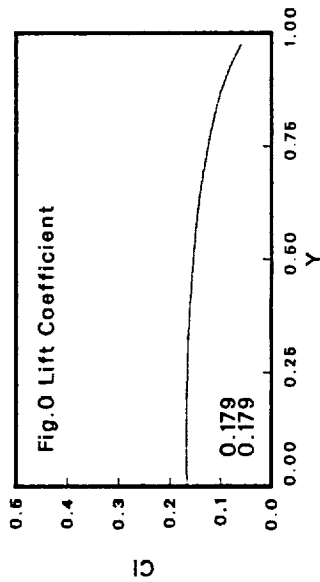


Fig.9 Sensitivity of Pressure Coefficient to Twist Angle at 4th Station





# TEST CASE 2

SUPERCRITICAL CASE

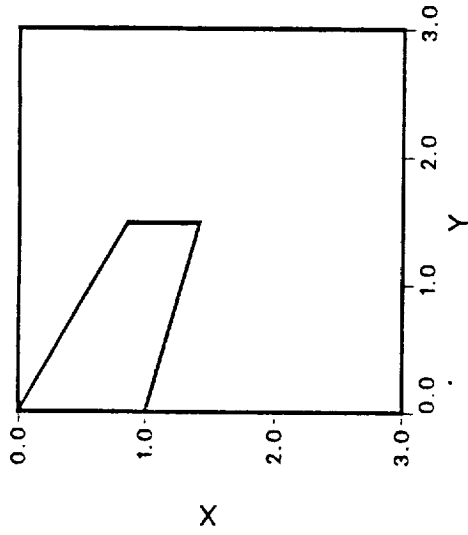
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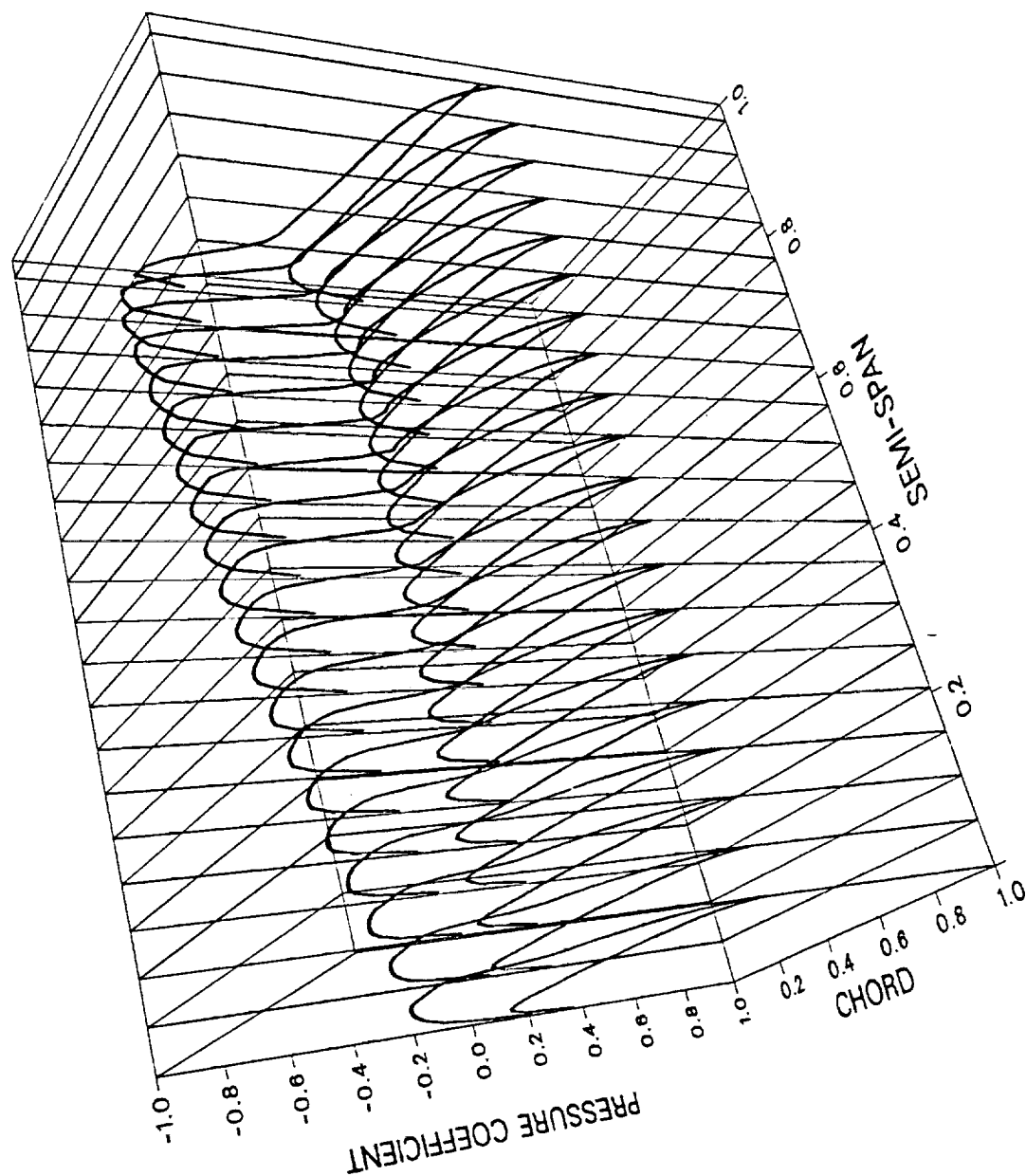
MACH NUMBER	0.84
ANGLE OF ATTACK	3.00
AIRFOIL MAX THICKNESS	0.06
AIRFOIL MAX CAMBER	0.01
LOCATION OF MAX CAMBER	0.40

# WING PLANFORM :

## ONERA M6



ROOT CHORD	1.00	ASPECT RATIO	3.80
TIP CHORD	0.56	TAPER RATIO	0.56
MEAN CHORD	0.80	SEMI SPAN	1.48
AREA	1.16	L.E. SWEEP	30.00
REF. AREA	1.16	T.E. SWEEP	15.76
REF. CHORD	0.80	ROOT TWIST	0.00
REF. MOMENT	0.25	TIP TWIST	0.00



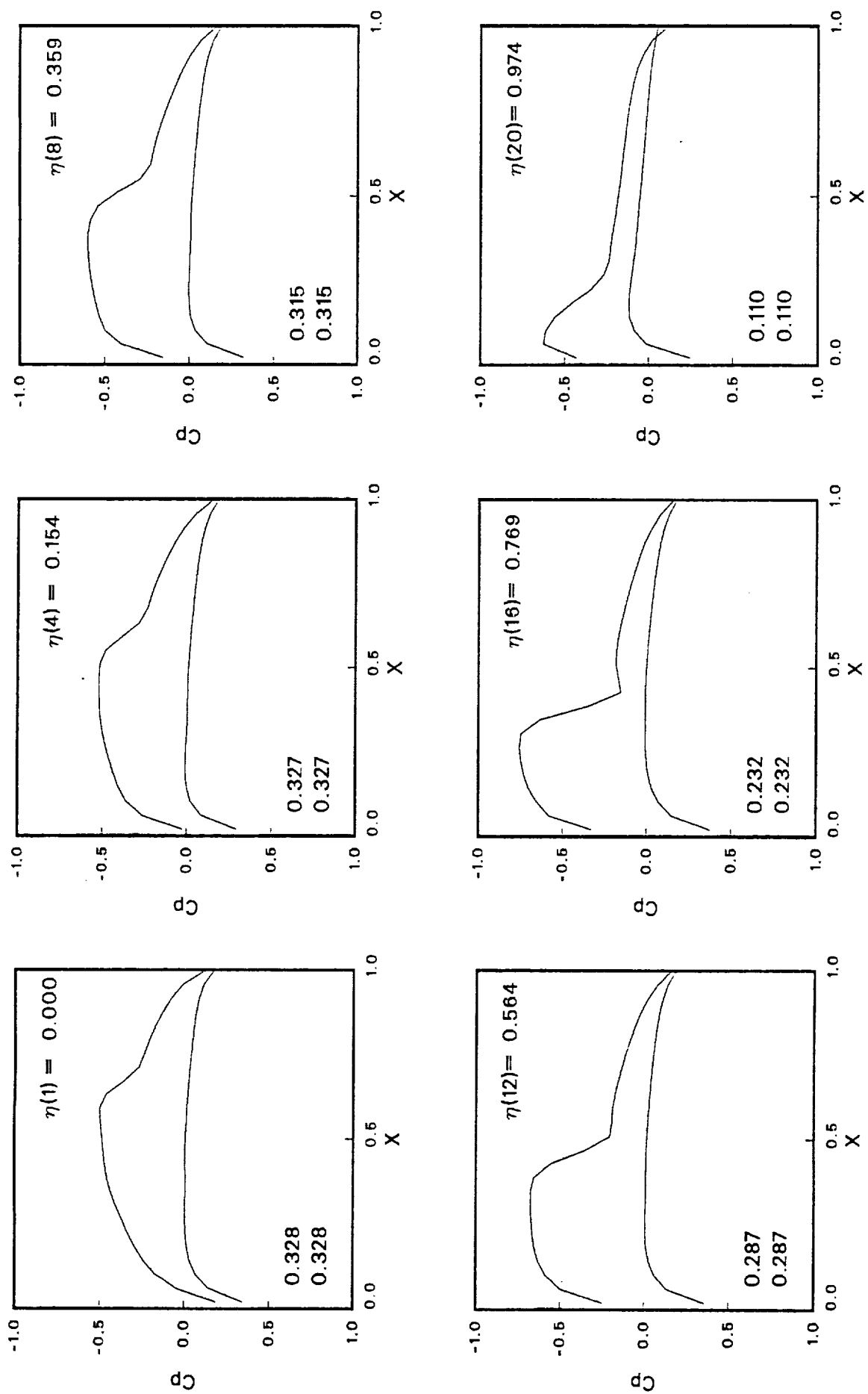


Fig.0 Pressure Coefficient

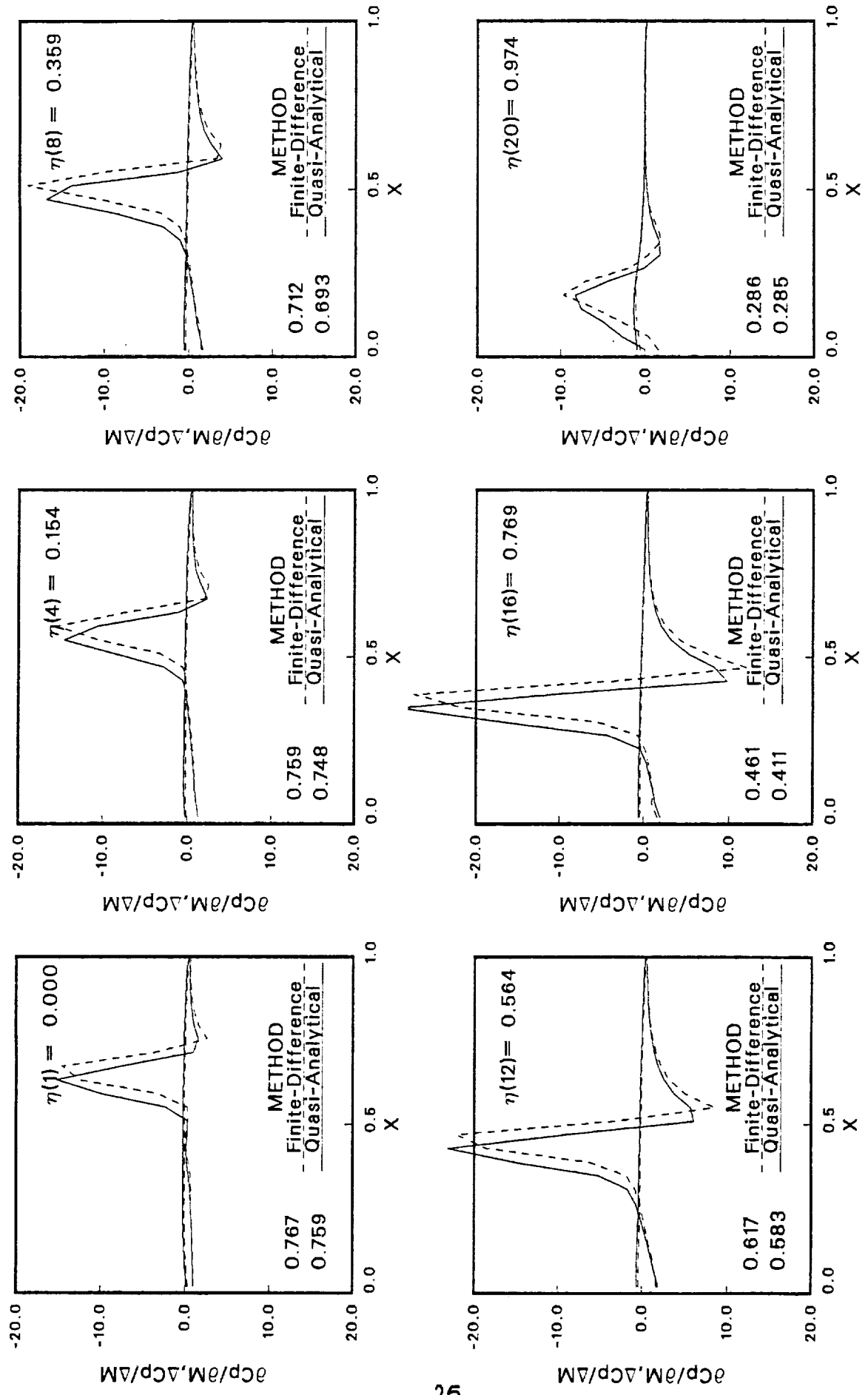


Fig.1 Sensitivity of Pressure Coefficient to Mach Number

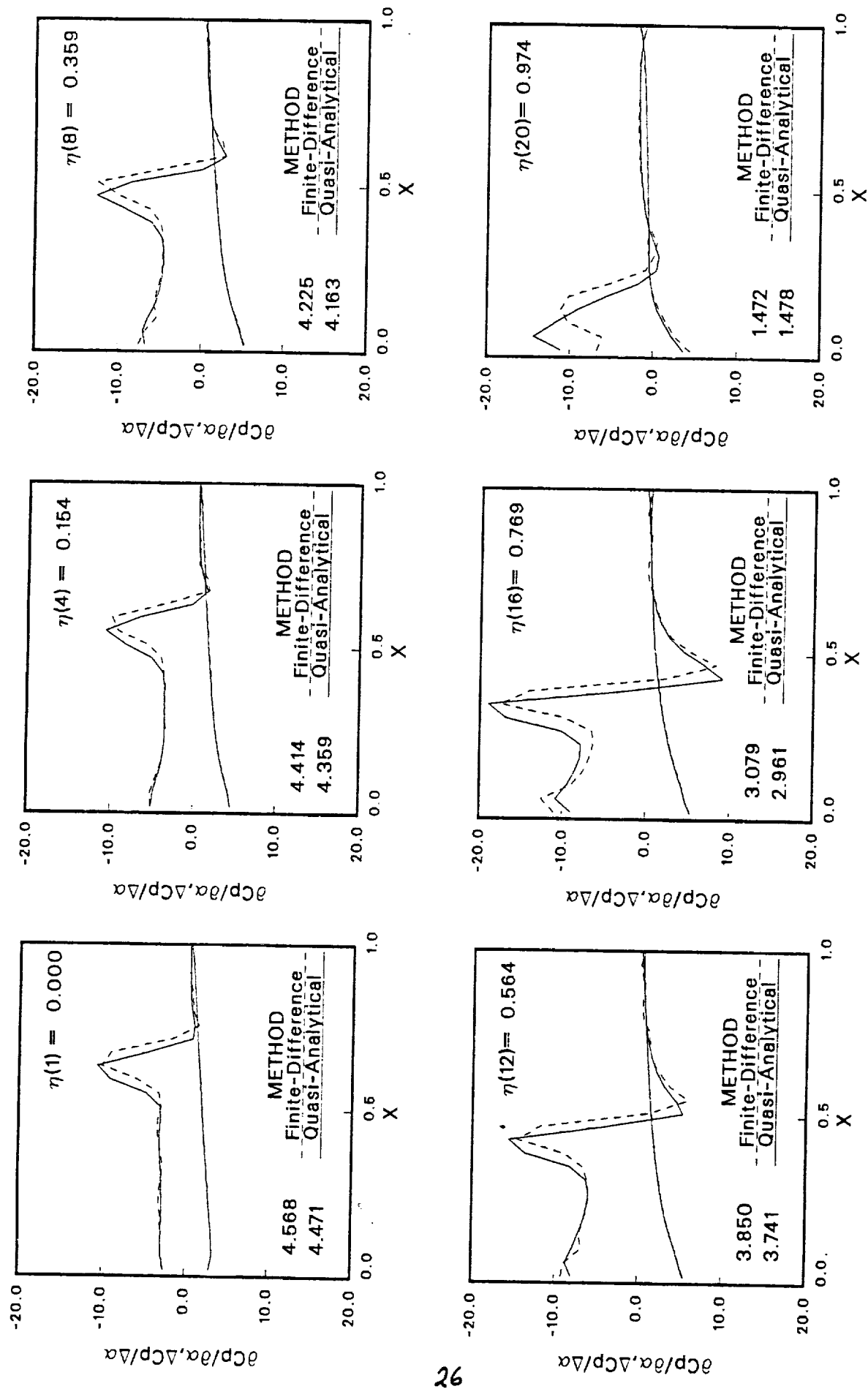


Fig.2 Sensitivity of Pressure Coefficient to Angle of Attack

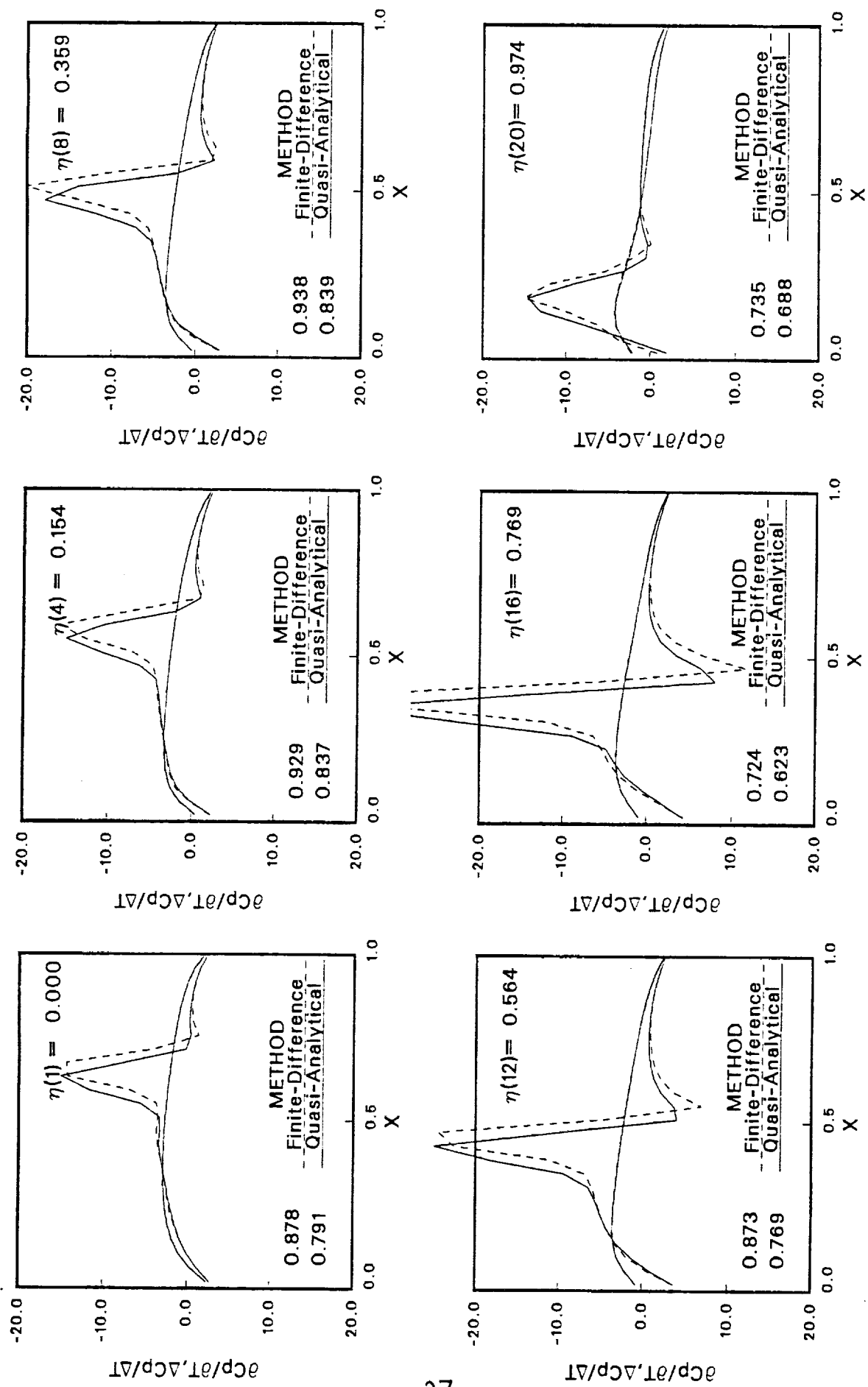


Fig.3 Sensitivity of Pressure Coefficient to Maximum Thickness



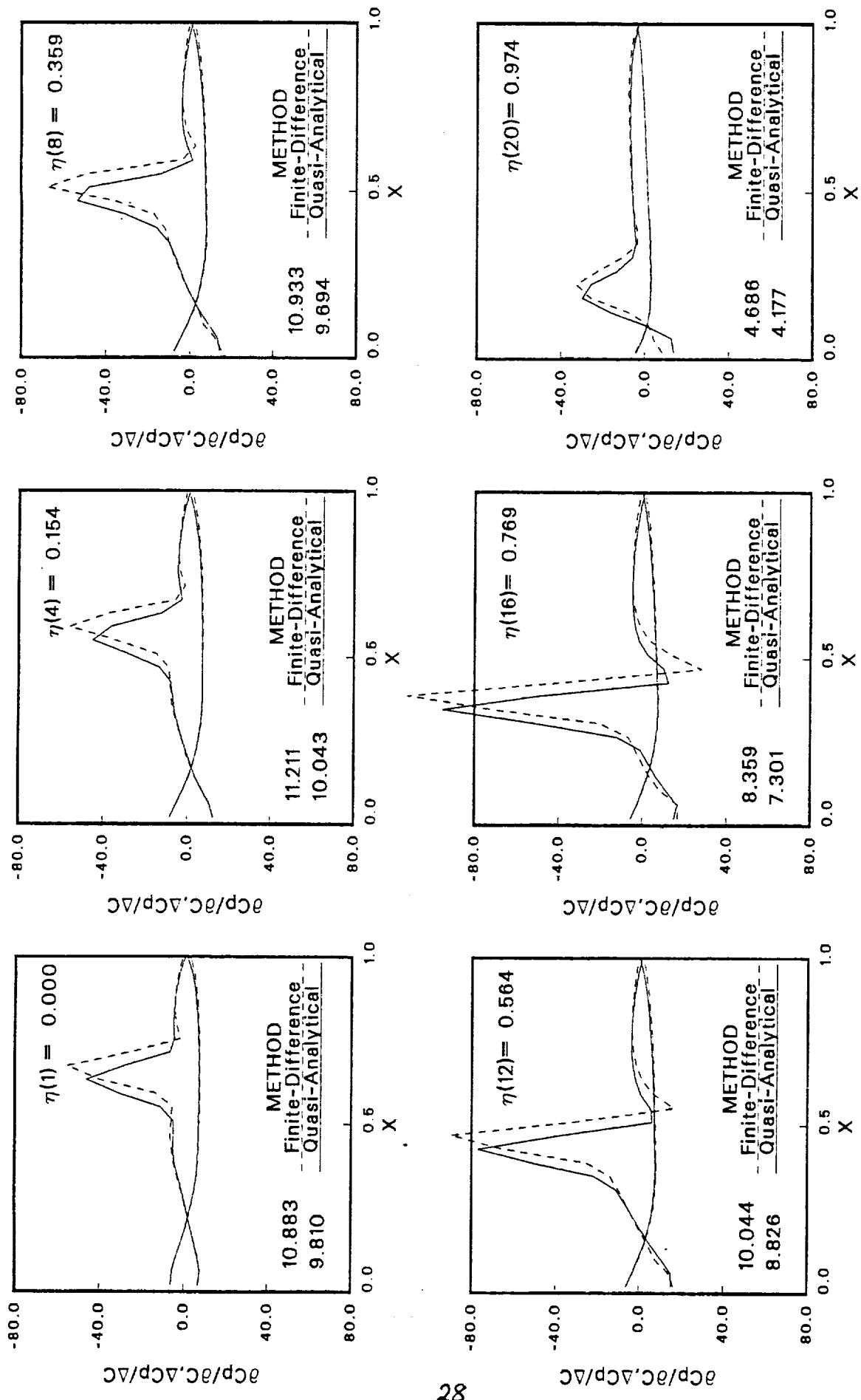


Fig.4 Sensitivity of Pressure Coefficient to Maximum Camber

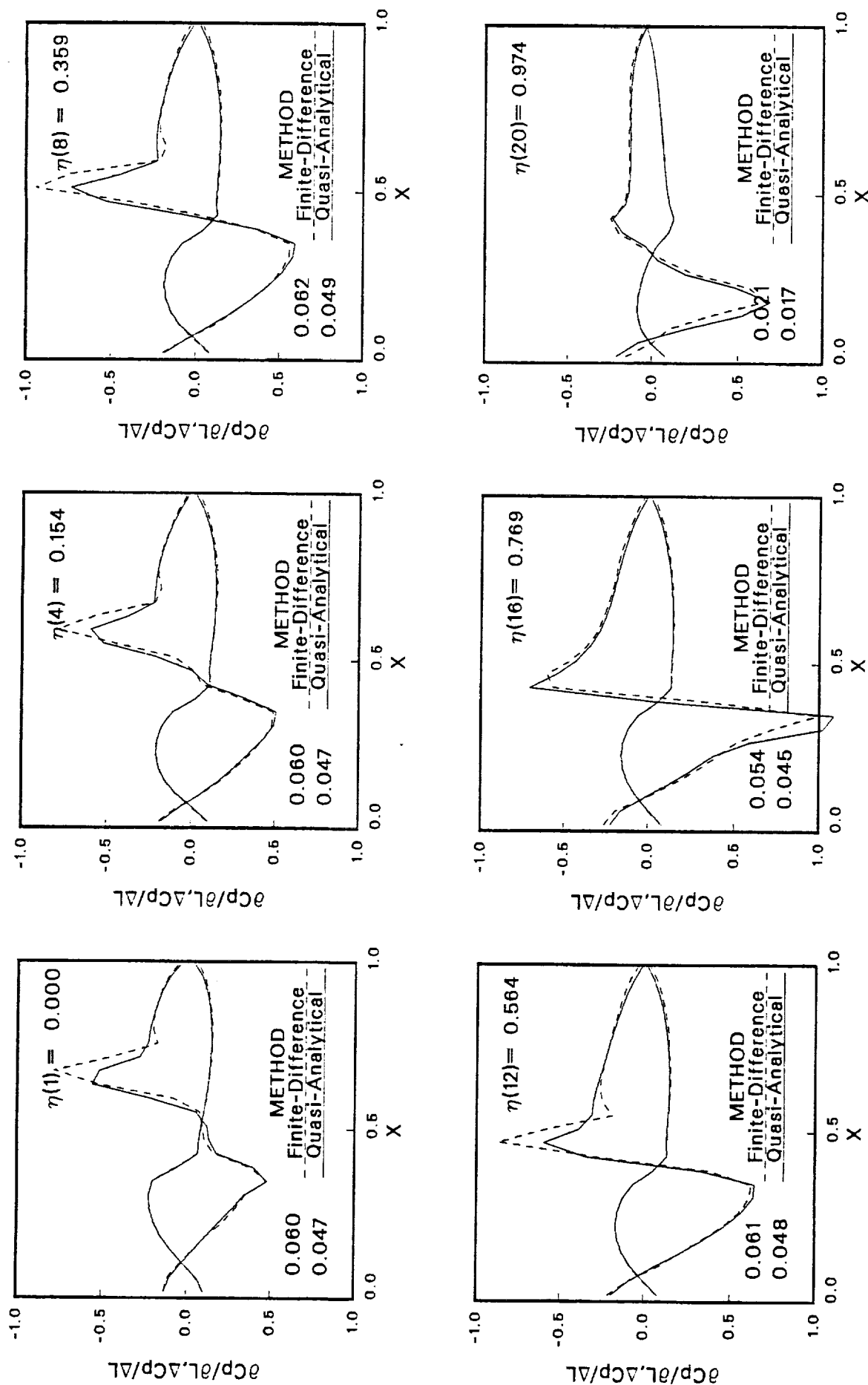


Fig.5 Sensitivity of Pressure Coefficient to Location of Maximum Camber

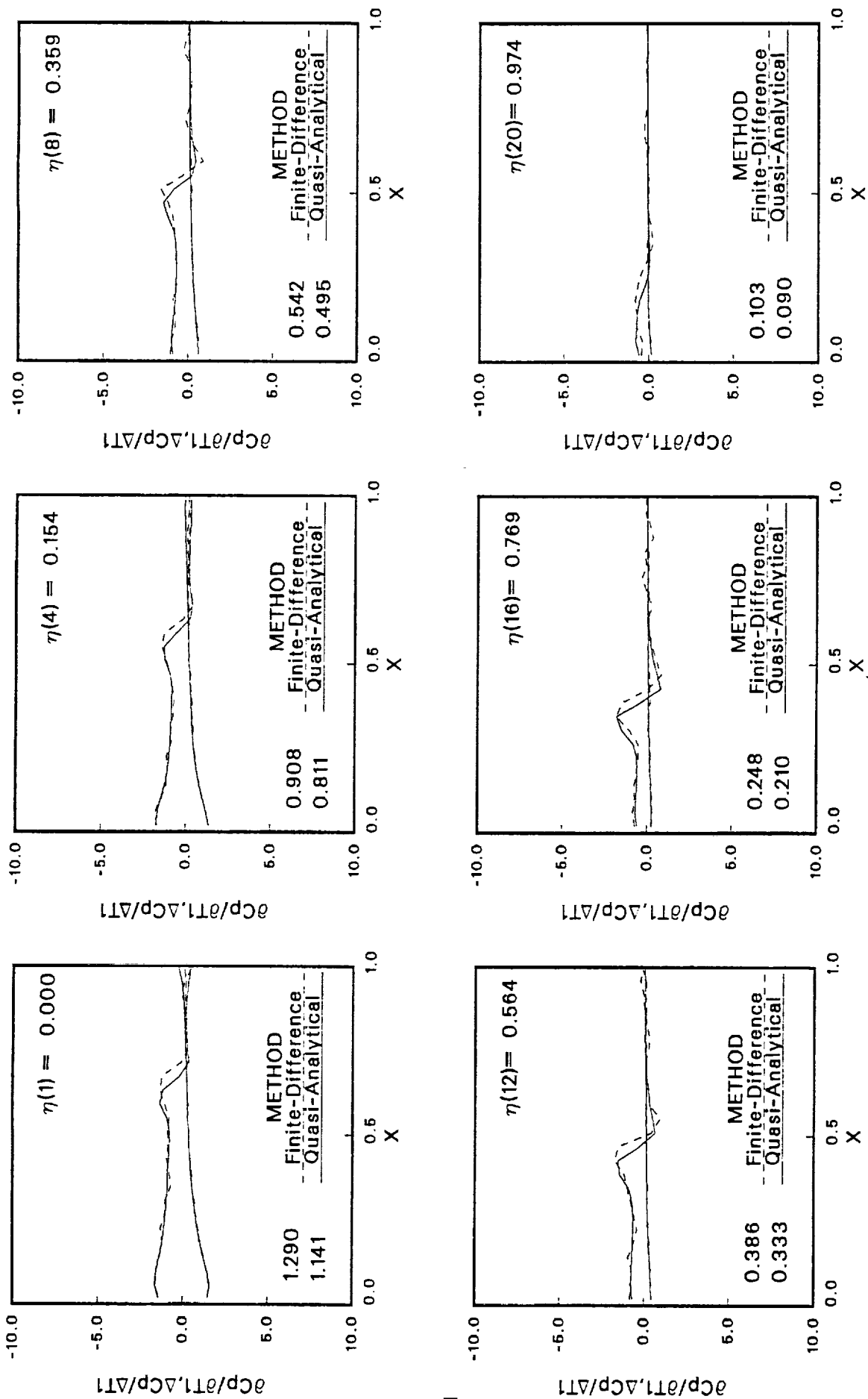


Fig.6 Sensitivity of Pressure Coefficient to Twist Angle at 1st Station

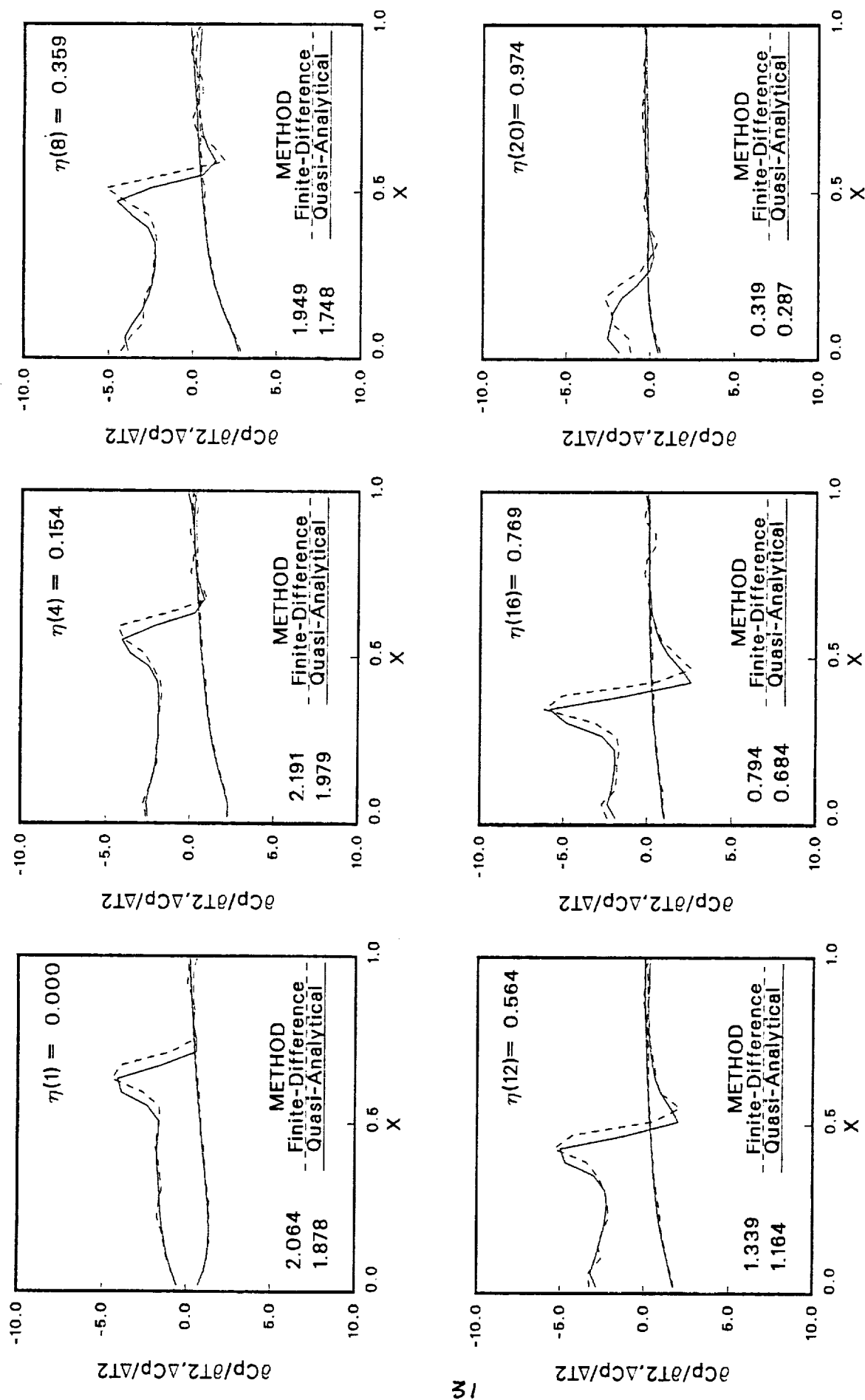


Fig.7 Sensitivity of Pressure Coefficient to Twist Angle at 2nd Station

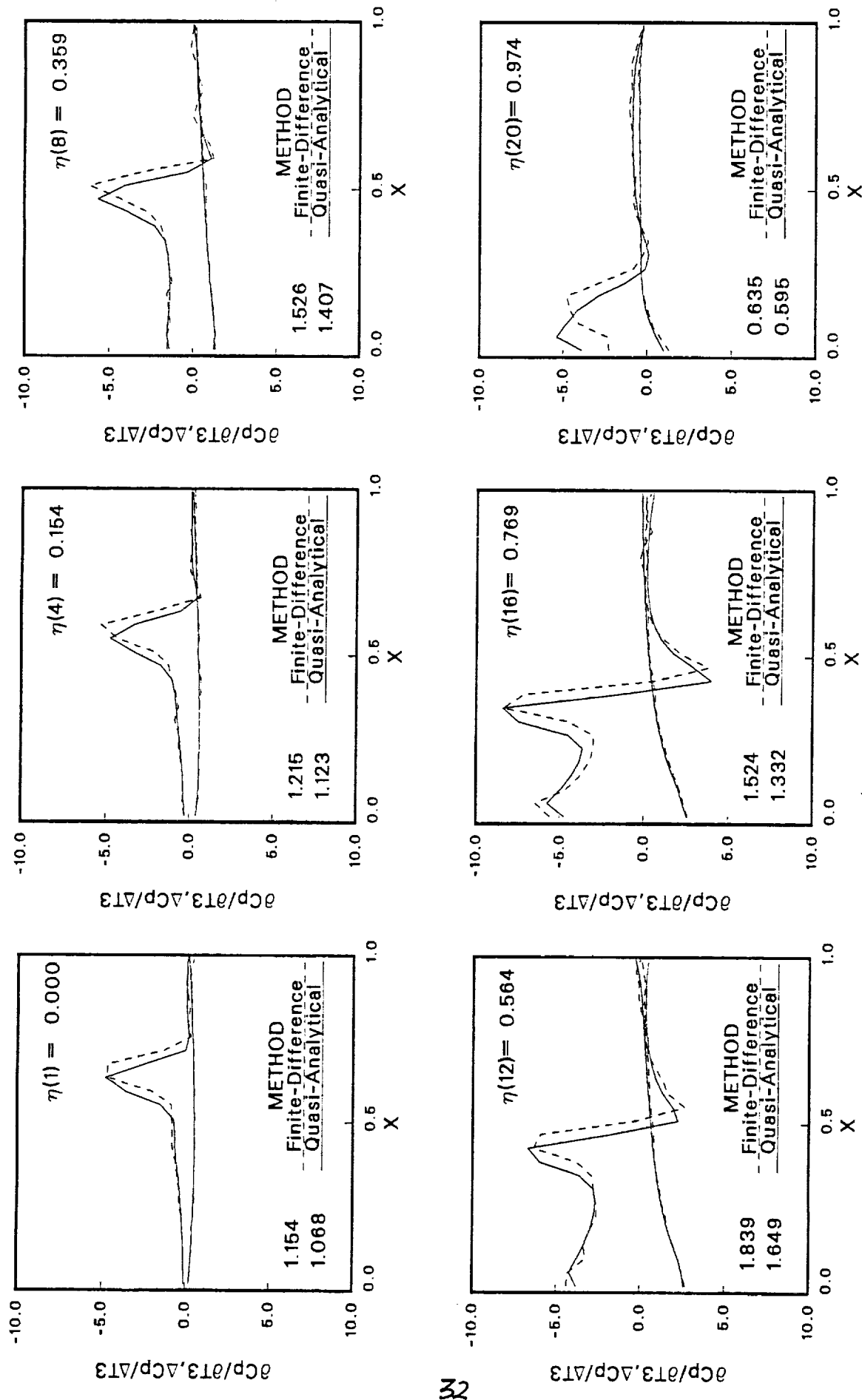


Fig.8 Sensitivity of Pressure Coefficient to Twist Angle at 3rd Station

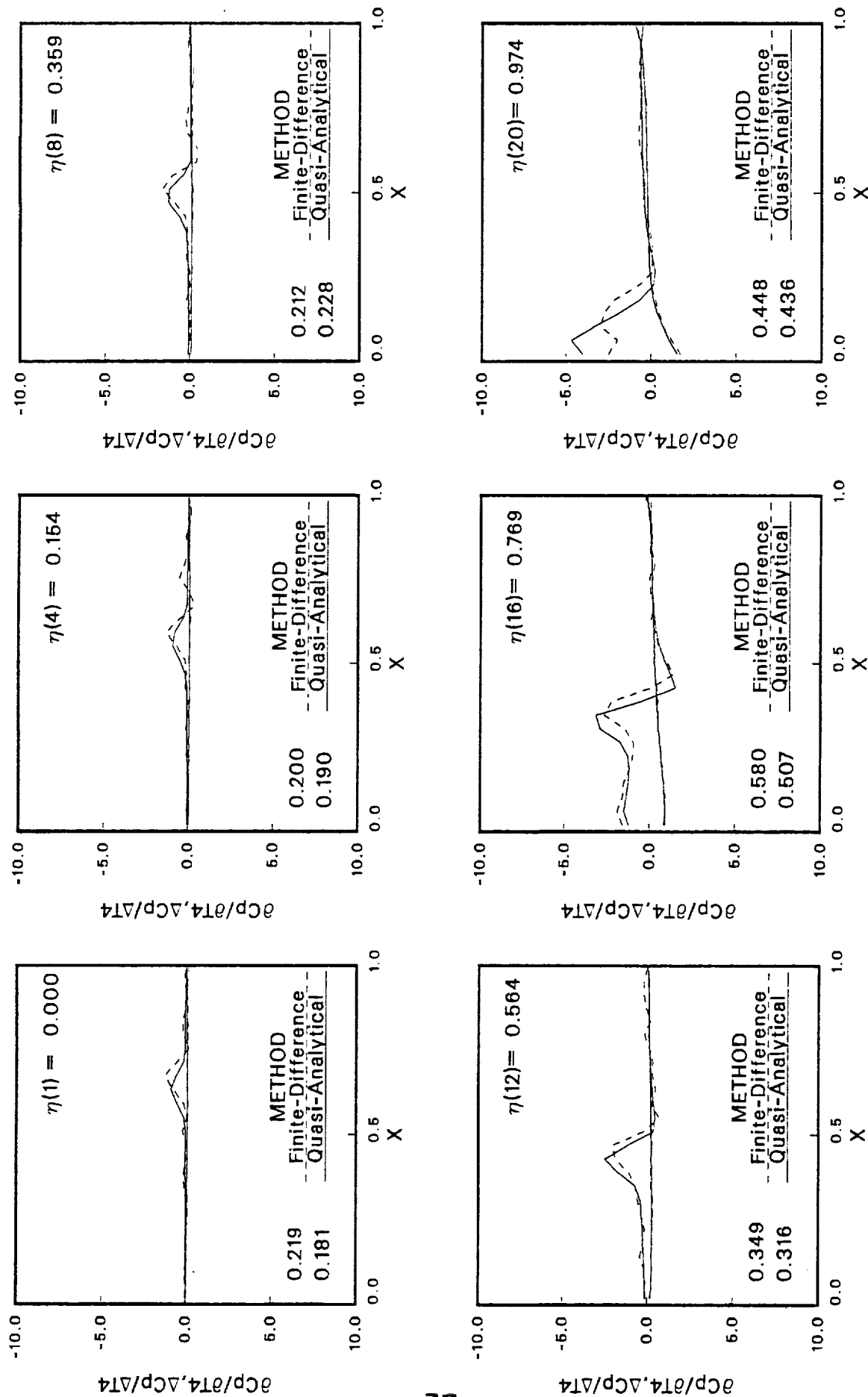


Fig.9 Sensitivity of Pressure Coefficient to Twist Angle at 4th Station

